STUDIES ON LONGITUDINAL COMPRESSION OF SPACE CHARGE DOMINATED BEAM

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Abstract

The longitudinal and transverse dynamics of space charge dominated beam have been studied self consistently during the bunch compression. The effect of axial variation of the longitudinal electric field has been considered in the transverse motion. We have also included a gaussian energy spread in the cw beam and studied its effect on the beam dynamics. We have studied the bunching behaviour of a sinusoidal buncher using the modified beam envelope equation for space charge dominated 100keV proton beam.

INTRODUCTION

The injection line of 10 MeV, 5mA compact cyclotron consists of a 2.45 GHz microwave ion source to deliver 100keV, 20mA proton beam, a low energy beam injection line with two solenoids, a sinusoidal buncher to bunch the beam suitably and a spiral inflector to place the injected beam on the proper orbit [1]. The typical value of phase acceptance of cyclotron is ~ 10 % of an rf cycle. Beam current in this phase acceptance can be improved by using a suitable buncher in the injection line. In most of the bunching systems the transverse beam dynamics are studied using envelope equation where the space charge force is taken proportional to the increase of beam current in the bunch during the bunch compression. This approximation is valid where bunch size is very large in comparison to the size of the beam radius and the variation of the line charge density is small. But when the bunch size becomes comparable to the beam radius, the axial variation of the longitudinal electric field cannot be neglected and the simple envelope equation is no longer valid. We have developed an appropriate envelope equation for the evolution of the bunch radius taking into account the effect of the variation of the longitudinal electric field. We have presented simulation results of a sinusoidal beam bunching system for various values of beam and buncher parameters.

METHOD

The longitudinal dynamics of the beam during the bunch compression has been studied using disc model where beam is divided into large number of discs in the beam frame. Each disc, identified by index i, is characterised by an axial velocity v_i and position z_i and contains a fixed charge Q and mass M. The equation of motion of each disc can be written as,

$$Mv \cdot \frac{dv_i}{ds} = F_i \text{ and } v \cdot \frac{dz_i}{ds} = v_i \qquad (1)$$

Here F_i is the total longitudinal force experienced by the i^{th} disc due to space charge and rf field. We have used Green function technique to calculate the average space charge field on a disc. The average electric field on the i^{th} disc due to j^{th} disc can be written as

$$\left\langle E_{ij}(z_i, s)\right\rangle = \frac{Q}{2\pi\varepsilon_0 \cdot R(s)^2} \cdot \sum_{n=1}^{\infty} \exp\left(-\beta_n \left|z_i - z_j\right|\right) \times \left(\frac{2 \cdot J_1\left(\beta_n \cdot R(s)\right)}{\alpha_n \cdot J_1(\alpha_n)}\right)^2 \cdot sign(z_i - z_j) \qquad (2)$$

where w and Q are the width and charge of each infinitesimally thin disc respectively. z_i and z_j are the positions of the i^{th} and j^{th} disc respectively. $R(s)$ is the radius of the disc at location s, and can be found by solving envelope equation. The average radial space charge field experienced by the bunch is given by

$$E_r(r, s) = \left(\frac{\left\langle \rho(s)\right\rangle}{\varepsilon_0} - \left\langle \frac{\partial E_z(0, s)}{\partial z}\right\rangle\right) \cdot \frac{r}{2} \qquad (3)$$

Using the expression of radial space charge field, the beam envelope equation can be written as [2]

$$R'' + k(s) \cdot R - \frac{\left\langle K_{eff}(s)\right\rangle}{R} - \frac{\varepsilon^2}{R^3} = 0 \qquad (4)$$

with $\left\langle K_{eff}(s)\right\rangle = \left\langle K(s)\right\rangle - \left\langle \Lambda(s)\right\rangle \cdot R^2$ (5a)

and $\left\langle \Lambda(s)\right\rangle = \frac{q}{m\beta^2\gamma^2c^2} \cdot \frac{1}{2} \cdot \left\langle \frac{\partial E_z(0, s)}{\partial z}\right\rangle$ (5b)

The term $\left\langle \Lambda(s)\right\rangle$ is the correction term in the radial force due to the axial variation of longitudinal electric field and $\left\langle K_{eff}(s)\right\rangle$ is the effective perveance of the beam at position s. $\left\langle K(s)\right\rangle = K_0 \cdot \left\langle I(s)\right\rangle / I$, is the average value of the perveance of the beam bunch. K_0, I are the perveance and current of the continuous beam respectively and $\langle I(s)\rangle$ is the average value of beam current of the bunch at location s during the bunch compression.

RESULTS AND DISCUSSIONS

We have chosen suitable drift length for each value beam current because of the restriction of maximum allowable drift length due to space charge effect for a given beam current [3]. Fig. 1 compares the variation of the effective perveance as a function of drift length with and without longitudinal part for 10 mA and zero energy spread in the beam.

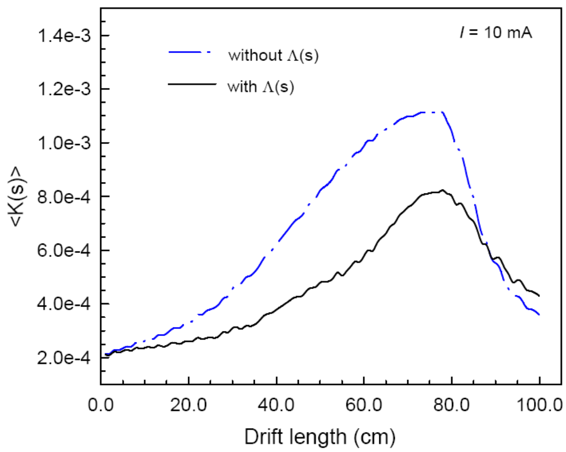

Figure 1. The variation of effective perveance <*K*(*s*)> of the beam bunch during compression for 100 keV protons.

The effect of bunch compression on the relative increase of beam current in the bunch <*I*(s)>/*I* is shown in Fig. 2 as the bunch travels along the drift space.

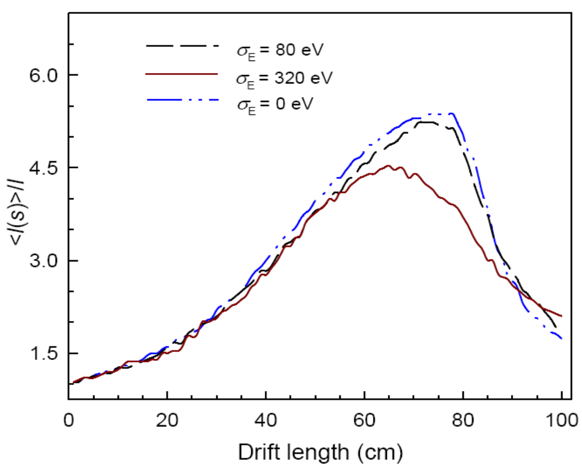

Figure 2. Evolution of <*I*(s)>/*I* during the longitudinal compression for 10 mA beam current at three different values of energy spread.

The effect of the energy spread on the bunching efficiency is plotted in Fig. 3 at two values of beam currents at 100 keV. As expected the energy spread reduces the bunching efficiency and effect is more dominant at higher beam currents. A phase space plot at the time focus is shown in Fig. 4. For low current, discs which are behind the bunch centre at buncher gap overtake discs which are ahead during the drift and cross through the bunch centre at the time focus and vice versa. However, for high beam current the motion of the discs are dominated by both rf and space charge field of the beam. The space charge force increases during the compression and act against the velocity modulation. As a result discs are repelled by the space charge force with reduction in number of discs crossing centre of bunch at the time focus.

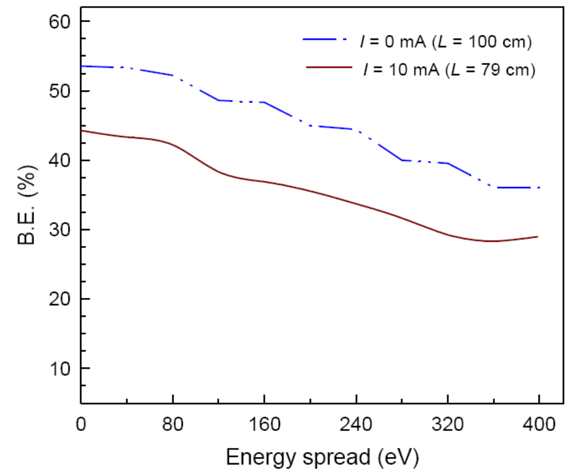

Figure 3. Variation of bunching efficiency (B.E.) as a function of energy spread at two different values of beam current at 100 keV.

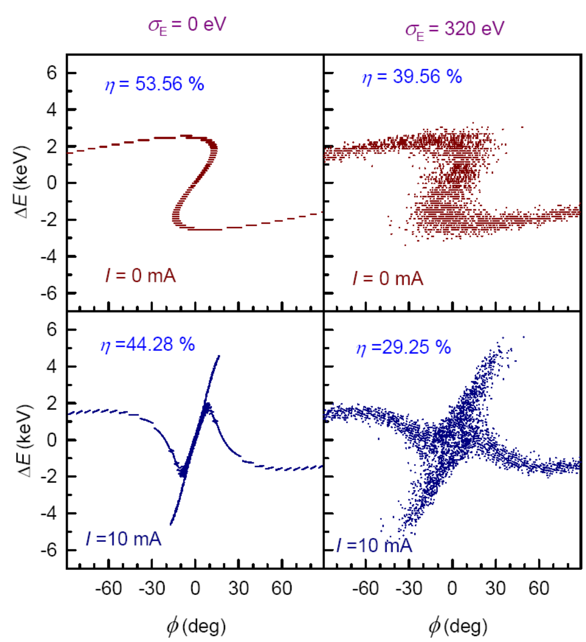

Figure 4. The longitudinal phase space distribution of the beam at the time focus for two different value of beam current and energy spread at 100 keV.

REFERENCES

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