emf generated by a magnetic dipole falling through a coil

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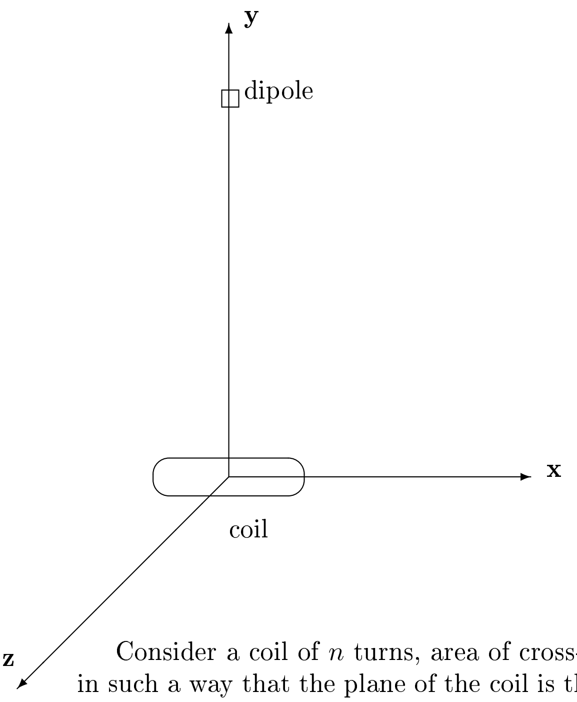
Theory



Consider a coil of n turns, area of cross-section A and negligible length. Let the axes be chosen in such a way that the plane of the coil is the x-z plane with the origin at the center of the coil. Let m be the dipole moment of a magnet that is dropped from a height h along -ve y-direction. Let the axis of the magnet be along y and g the acceleration due to gravity. Faradays law along with Lenz's law gives the induced emf as

$$\epsilon = -\frac{d\phi}{dt} = -nA\frac{db}{dt} \tag{1}$$

$$= -\frac{\mu_0\ 2mnA}{4\pi}\frac{d}{dt}\left(\frac{1}{y^3}\right) \tag{2}$$

$$= \frac{3\mu_0\ mnA}{2\pi\ y^4}\frac{dy}{dt} \tag{3}$$

$$= \frac{3\mu_0\ mnA}{2\pi\ y^4}\sqrt{2g(h-y)} \tag{4}$$

This expression gives emf as a function of distance of the magnet from the coil.

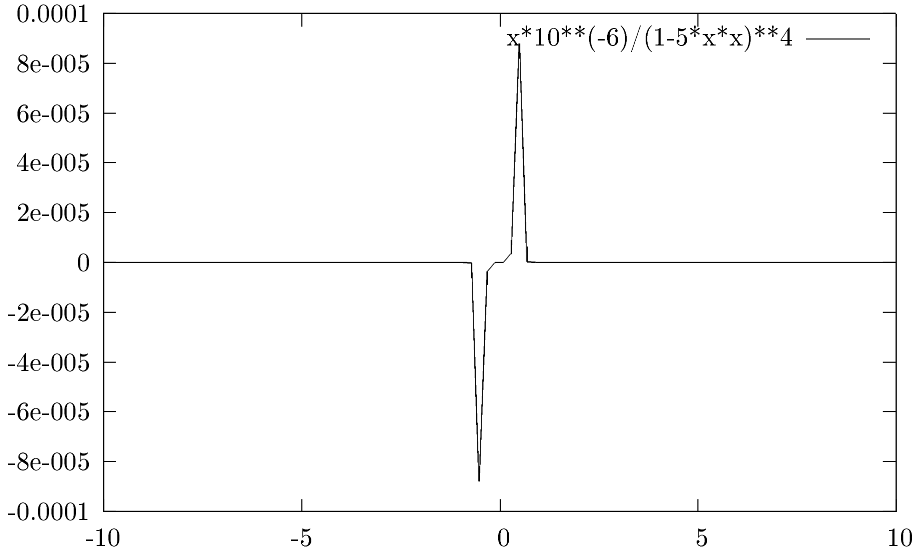
To get an expression for emf as a function of time, express y as a function of time.

$$h - y = \frac{gt^2}{2} \tag{5}$$

$$\epsilon = \frac{3\mu_0\, mnA}{2\pi\,(h - gt^2/2)^4}\sqrt{2g(gt^2/2)} \tag{6}$$

$$= \frac{3\mu_0\, mnA}{2\pi}\frac{gt}{(h - gt^2/2)^4} \tag{7}$$

The graph given below is for $g = 10$, $\frac{3\mu_0\, mnA}{2\pi} = 10^{-7}$, $h = 1$ and plotted using GNUPLOT



Approximations employed

Neglected quantities

1. The air resistance
2. The length of dipole
3. Linear dimension of the coil
4. The reduction of acceleration from g due to inductive reaction as electrical potential is developed at the expense of the kinetic energy of the falling magnet.
5. The dipole is assumed to fall in the direction of its dipole moment, no oblique fall considered.
6. The effects due to the relative size of the coil(radius of the coil) and size of the dipole (cross-sectional area)